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Sustainable Endogenous Growth and Consumption Inertia: Old habits die hard

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Abstract:

In this paper we study an endogenous growth model with habit-formation and address two questions that are, to the best of our knowledge, new for the sustainable endogenous growth literature: first, does the process of habit-formation in‡uence the stock of environmental capital? Second, does habit-formation a¤ects the long-term rate of economic growth?

Using a simple and standard structure of the endogenous growth models, we first show that there may be multiple equilibria, not all stable. Second, the presence of habits in relation to the consumption goods lowers the long term equilibrium level of natural capital and the growth rate of the economy. Third, we highlight the possibility of "win-win" situations.

Finally, we show that the presence of habits reduces the effectiveness of any environmental policy that is meant to improve environmental quality. In particular, the stronger the inertia effect, the lesser will be the equilibrium levels of natural capital and the greater will be the net flow of pollutant emissions. At the same time, the economy will grow at a lower rate.

Keywords: Endogenous Growth, Sustainability, Environmental Preservation, Habit- Formation.

JEL Classification: C61, D11, D90, Q21

1 Introduction

The formal literature on sustainability tends to adopt a very long run perspective. One of the assumptions consistent with that perspective is that households' preferences are additively separable or, equivalently, intertemporally independent. This implies that consumer demand may (optimally) shift substantially for a change in preferences, technology, or policy incentives. Ryder and Heal (1973) were the first to show that the presence of intertemporal dependent preferences might be a sufficient reason to cause a cyclical behavior of consumption along its optimal path.

To the best of our knowledge, the process of intertemporal dependent preferences and its influence over consumer (and consumption) behaviour has not yet been fully developed and studied in Environmental Economics. Sustainability, defined as a non-decreasing welfare over time (Pearce and Turner (1990), Pearce et al. (1990), Titenberg (2003) or Perman et al. (2003)) has been studied in an intertemporal independence framework. However, if preferences are intertemporal dependent the possible resulting cyclical behavior of consumption along optimal path might jeopardize the altruistic dimension (equity) (Pearce and Turner (1990)) of sustainability, at least during some moments in time.

On the other hand, it has been generally accepted that consumption adjusts substantially quickly after an exogenous shock. This has been crucial for proving sustainability but is possibly unrealistic. Consumption might exhibit some degree of inertia and take some time to adjust to a new optimal time path after an exogenous stimulus toward, for example, a more green economic behavior.

Finally, given the civilizational and cultural contour of preferences towards the environment we may expect a protracted short to medium run adjustment of consumption and of related conservationist policies. Again, this adjustment might be influenced by the inertia that characterizes consumption adjustment due to the intertemporally dependent preferences.

This paper addresses two questions that are, as far as we know, new for the environmental and resource economics: first, does the process of habit-formation influence the stock of environmental capital? Second, does habit-formation affect the long term rate of economic growth?

Löfgren (2002) is, to the best of our knowledge, the sole reference in the literature devoted to sustainability who analyses the influence of habit-formation over the optimal allocation between the consumption good and the environment. Using a simple ECO-ECO model where production is totally consumed and the instantaneous utility function is log-linear in all its arguments, his major finding is that the presence of habit formation is neutral in the sense that it has no effect over the optimal level of environmental quality.

There are several ways to build intertemporal dependent preferences. In the recent literature on macroeconomics (both closed and open) and finance the Ryder and Heal (1973) model for the so-called habit formation has been used to solve a number of puzzles. And it conforms well with the short run low volatility of consumption. Intuitively, it says that the consumer alongside with current consumer builds a stock of habits. This stock of habits can be thought as the weighted sum of the history of past consumption and it is responsible for the introduction of some degree of inertia in current consumption.

In this paper we use the most simple and flexible structure by the consideration of a centralized economy where consumers have intertemporally interdependent preferences only as regards the consumption of material goods. Production is modelled to exhibit constant returns of scale and where "economic capital" is seen "à la Rebelo". Instantaneous utility is assumed to be homogeneous and the stock of habits is understood as a "beneficial" addiction, rather than one that is "bad", as it is often used in the literature (see Carroll (2000)).

Our preliminary conclusions are the following: first, we show that there may be zero, one or two equilibria, not all stable. Second, the presence of habits in relation to the consumption goods lowers the long term equilibrium level of natural capital and the growth rate of the economy. Third, we highlight the possibility for "win-win" situations, that is, an increase in environmental quality and an increase in the long term growth rate as a result of a change of environmental preferences towards a more environmental concern. Finally, we show that the presence of habits in relation to the consumption of manufactured goods influences the effects of the changes of the amenity value of the environment over the existing levels of natural capital, pollutant emissions and on the long term growth rate. In particular, the stronger the inertia effect caused by habits is, the lower will be the equilibrium level of natural capital and the long-run growth rate and, the higher will be the net flow of pollutant emissions as a result of an environmental policy that shifts preferences towards more concerns for a clean environment.

The paper is organized as follows. In section one, we present and develop the basic framework of the model with special emphasis on the structure of intertemporal preferences due to the presence of habit persistence. The next section is devoted to the determination of the conditions under which the model generates a balanced growth path. We also make some comparative static analysis and section 4 concludes the paper.

2 The general structure of preferences

Consider the problem in which the current well being is determined by current flow of consumption goods, C, by "habit stock" determined by past consumption H, and by the services generated by natural capital, N. The instantaneous utility function takes the following specific form:

$$u\left[C(t), H(t), N(t)\right] = \frac{\left[(CH^{\varphi})^{\alpha} N^{\beta}\right]^{1-\sigma}}{1-\sigma} \tag{1}$$

where σ is the well known coefficient of relative risk aversion of the standard CRRA model and it is assumed to verify the usual condition $0 < \sigma < 1$.

On the other hand, α, β and φ indexes the importance of manufactured goods, natural capital and habits over instantaneous utility. The first two parameters are assumed to be positive; $\alpha > 0$ and $\beta > 0$.

If $\varphi = 0$ then only the current absolute level of consumption and the services provided by natural capital are important and we have the standard CRRA model. But if $\varphi \neq 0$, then both current and past consumption levels (as well as natural capital) are relevant for present well-being. In other words, the instantaneous well-being is determined not only by the instantaneous level of consumption (the *level effect)* but also by its (average) past level throughout a process of "learning-by-consuming" (the *habit* or *persistence effect*).

In particular and differently to the main stream of the habit-formation literature (see, for example, Ryder and Heal (1973), Constantinides (1990), Carroll (2000), Wendner (2000)), we follow Becker and Murphy (1988) by assuming that the stock of habits has a positive value for consumer. To use Becker and Murphy (1988)'s words, past consumption level is viewed as a "beneficial" addiction which means that the more the past level of consumption, the less is required to derive the same level of utility in the present, for any given current level of consumption and of natural capital. This implies that $\varphi > 0$[1].

Finally, in order to assure the existence of a Balanced Growth Path (BGP), the instantaneous utility function is homogeneous in degree $(1-\sigma)\left[\alpha\left(1+\varphi\right)+\beta\right] < 1$ and concave in respect to all arguments.

Let the paths of consumption of manufactured goods, of the natural capital and of the stock of habits in relation to the consumption of manufactured good be denoted by $C := \{C(t) : 0 \leq t < \infty\}$, $N := \{N(t) : 0 \leq t < \infty\}$ and $H := \{C(t) : -\infty < t \leq 0\}$. The intertemporal utility function is the functional over the instantaneous flows of utility, discounted at the positive and constant rate δ,

$$V(C,H,N) = \int_t^{\infty} u\left[C(t), H(t), N(t)\right] e^{-\delta t} dt \tag{2}$$

As we saw previously, the novel feature of the utility is the presence of habits in relation to the consumption of manufactured goods. These habits are modeled as an exponentially weighted sum of past aggregate consumption levels[2]:

$$H(t) = H(0)e^{-\rho t} + \rho \int_0^t e^{-\rho(t-s)} C(s) ds \tag{3}$$

where the coefficient $\rho > 0$ measures both the rate of decay of the habits and the rate of habit formation from the current flow of services provided by consumption of manufactured goods. This equation may be written in its differential form, showing how the stock of habits evolves.

$$\dot{H}(t) = \rho\left[C(t) - H(t)\right] \tag{4}$$

[1] Or, to be more precise, $u_H > 0$.

[2] This specific form of habit formation is what some literature calls the "*inward-looking*" habit formation process. If, instead, the stock of habits is formed using other's consumption level, then it is said to be generated by an "*outward-looking*" process. See, for example Carrol et al. (1997).

In this form, the parameter ρ can also be seen as denoting the strength of habits. The larger the value of this parameter, the more important is consumption in the recent past.

The utility functional defined by (2) and (3) models the presence of complementarity over time in consumption of manufactured goods, in the sense that any unitary change of consumption at time $t = t_0$ has not only an impact on current utility given by the instantaneous marginal utility, but also an impact over the future level of utility, measured by the intertemporal marginal utility. More formally, if a given path of consumption good $\{C(t)\}$ is perturbed at an arbitrary time $t = t_0$, then the resulting change in the value of the functional (2) over time can be captured by the following Volterra derivative, which is itself another functional.

$$V_C(.)\delta(t_0) = u_C e^{-\delta t_0} + \rho e^{\rho t_0} \int_{t_0}^{\infty} u_H(t_0) e^{-(\rho+\delta)t} dt \tag{5}$$

where $\delta(t_0)$ is the Dirac's delta. If, for simplicity, we assume stationary perturbation over the steady state level of consumption $\overline{C} = \{C(t) = \overline{C}, 0 \leq t < \infty\}$ and of the natural capital $\overline{N} = \{N(t) = \overline{N}, 0 \leq t < \infty\}$, then the two intertemporal marginal utilities of a permanent shift in consumption at any time are the sum of the present value of the correspondent instantaneous marginal utilities[3]

$$V_C(.)\delta(t_0) = e^{-\delta t_0}\left[u_C + \frac{\rho}{\rho+\delta}u_H\right] > 0 \tag{6}$$

$$V_N(.)\delta(t_0) = u_N e^{-\delta t_0} > 0 \tag{7}$$

Given the properties of the utility, these expressions also show that there is no satiation in the consumption of the two goods. The marginal rate of intertemporal substitution between natural capital and consumption goods in then given by

$$MRIS_{N,C} = -\frac{u_N}{u_C + \frac{\rho}{\rho+\delta}u_H} \tag{8}$$

This magnitude is, in absolute value, smaller then its correspondent without the "inertia effect" of consumption. Due to the presence of habits, any increase of consumption needs to be compensated by a higher reduction in the level of the natural capital in order to assure the maintenance of the intertemporal flow of utility. This "overweight"

[3]The intertemporal marginal utility can also be measured by the Fréchet derivative of the functional (2). In this case, we would get a similar expression, in which the discount factor would be replaced by the infinite sum of discounted factor $\frac{1}{\delta} = \int_0^{\infty} e^{-\delta t} dt$.

of consumption in utility is also evident when the converse exercise is made: an increase of the natural capital will be compensated by a lesser reduction of the consumption in order to maintain constant the flow of well-being.

The marginal rate of intertemporal substitution between consumption at moments t_0 and $t_1 > t_0$ is the ratio of the marginal utilities $R(t_0,t_1) = \frac{V_C(.)\delta(t_0)}{V_C(.)\delta(t_1)}$ (see Ryder and Heal (1973)) and the intertemporal dependencies are measured by the change of this ratio as a result of a change of consumption at time $t_2 > t_1 > t_0$,

$$R_C(t_0,t_1;t_2) = \frac{V_C(.)\delta(t_1)V_{CC}(.)\delta(t_0)\delta(t_2) - V_C(.)\delta(t_0)V_{CC}(.)\delta(t_1)\delta(t_2)}{[V_C(.)\delta(t_1)]^2} \tag{9}$$

where $V_{CC}(.)\delta(t_i)$ is the second order Volterra derivative in order to C at t_i;

$$V_{CC}(.)\delta(t_0) = \rho u_{CH} e^{-(\rho+\delta)t_1+\rho t_0} + \rho^2 e^{\rho(t_0+t_1)} \int_{t_1}^{\infty} u_{HH} e^{-(2\rho+\delta)t} dt \tag{10}$$

Again, without any lost of generality, along a stationary solution, we can write this expression as;

$$V_{CC}(.)\delta(t_0) = \rho e^{-(\rho+\delta)t_1+\rho t_0} \left[u_{CH} + \frac{\rho}{2\rho+\delta} u_{HH} \right] \tag{11}$$

Substituting 11 and 6 into 9, we obtain

$$R_C(t_0,t_1;t_2) = \rho \left[\frac{u_{CH} + \frac{\rho}{2\rho+\delta} u_{HH}}{u_C + \frac{\rho}{\rho+\delta} u_H} \right] \Delta \tag{12}$$

where $\Delta = e^{-\delta(t_1-t_0)} \left[e^{(\rho+\delta)(t_0-t_2)} - e^{(\rho+\delta)(t_1-t_2)} \right] < 0$ as we have $t_2 > t_1 > t_0$.

If $R_C(t_0,t_1;t_2) < 0$, then it is said that there is adjacent complementarity[4] which, given the concave properties of the utility function, only occurs when

$$u_{CH} > -\frac{\rho}{2\rho+\delta} u_{HH} > 0 \tag{13}$$

This means that past consumption levels of manufactured goods raises the marginal utility of present consumption[5].

[4] There is adjacent complementarity when a unit increment in consumption at t_2 shifts consumption from t_0 to t_1, with $t_0 < t_1 < t_2$. In other words, increases in consumption in the present are positively auto-correlated with recent increases in consumption, as opposed to distant increases, in time. See Ryder and Heal (1973).

[5] It should be noted that "adjacent complementarity" should not be identified with "addition". To use

3 Technology and the environment

We assume an economy that produces a single and homogenous good by using both "economic" and natural capital stock as production factors. Economic capital should be interpreted as a broad sense of capital as in Rebelo (1991), including, not only physical capital but also human capital and technical knowledge.

This homogenous good can either be consumed, invested and/or even used for pollution abatement $A(t)$. Formally,

$$\dot{K}(t) = Y(t) - C(t) - A(t) \tag{14}$$

Furthermore, it is assumed that this economy has convex technology in the specific form of a linear production function regarding physical capital stock,,

$$Y(t) = F\left[N(t), K(t)\right] = BN^{\varepsilon}K \tag{15}$$

where B is a scale parameter which can be assumed to be unity[6].

The specific form of this production function implies that along the balanced growth path, if it exists, the growth rate is dependent on the optimal level of environmental asset (which is constant) and on the shares of consumption and abatement expenditures in the capital stock. Using 14 it is easy to see that $\gamma_K = \gamma_C = \gamma_A = B\left(N^*\right)^* - c^* - a^*$, where $c^* = \frac{C^*}{K^*}$ and $a^* = \frac{A^*}{K^*}$ are the correspondent steady state values.

The productive sector is responsible for pollution emissions that affects the regenerative and assimilative capacity of the natural capital. This flow of emission is assumed to be proportional to the man-made capital stock.

However, not all the emissions are discharged into natural system since the economy devotes part of his income to pollution-abatement activities in order to reduce environmental damages. Then the net flow of pollution that ends in natural system at any time t is

$$P(t) = p\left[K(t), A(t)\right] = \frac{1}{a(t)} = \frac{K(t)}{A(t)} \tag{16}$$

Finally, we assume that the environmental asset has the natural ability not only to renew itself but also to assimilate and regenerate part of the pollution emissions

Becker and Murphy (1988)' words, "*...a person is addicted to c if an increase in his current consumption of c increases his future consumption of c*". Of course, for addiction it is a necessary but not sufficient to have adjacent complementarity.

[6] From now on we will ignore any reference to time unless it is absolutely relevant and necessary. This also means that it will be assumed that all variables evolve through time, unless the opposite is stated.

generated by production. We model this regenerative capacity as a growth and depletion process of a renewable resource;

$$\dot{N}_n = n(N) = \theta N \left[1 - \frac{N}{N_{CC}}\right] \tag{17}$$

where $\theta > 0$ and $N_{CC} > 0$ are, respectively, the *intrinsic natural growth rate* and the *natural carrying capacity,* the maximal stock of natural capital that can be kept intact only by natural regeneration. This formulation acknowledges the presence of diminishing returns in environmental processes due to the law of thermodynamics and, specifically, the law of entropy. For low levels of environmental quality, the regenerative and assimilative capacity is increasing with the capital stock but it decreases when the natural capital stock becomes large. In other words, the higher the stock of natural capital, the more difficult it is to regenerate the complete stock, given the (almost) fixed level of solar inflow to earth;

$$n_N > 0 \quad \text{if} \quad N < N_M \quad \text{but for } N > N_M \ ; n_N < 0 \tag{18}$$

So, the net change of the stock of environmental capital is given by

$$\dot{N} = n(N) - \frac{1}{a} \tag{19}$$

The intertemporal optimization problem for the centralized version of this economy is

$$\max_{C(t)} \int_0^\infty \frac{\left[(CH^\varphi)^\alpha N^\beta\right]^{1-\sigma}}{1-\sigma} e^{-\delta t} dt \tag{20}$$

subject to the three equations of motion (14), (19) and (4) and given $H(0) = H_0$, $K(0) = K_0$ and $N(0) = N_0$. In the next sections we determine the (optimal) balanced growth path.

4 The balanced growth path

The balanced growth path, is defined by the paths of consumption, abatement activities, the stock of man-made capital, the stock of habits and of the stock of natural resources, $\{\{\overline{C}(t)\}_{t=0}^\infty, \{\overline{A}(t)\}_{t=0}^\infty, \{\overline{K}(t)\}_{t=0}^\infty, \{\overline{H}(t)\}_{t=0}^\infty \{\overline{N}(t)\}_{t=0}^\infty\}$, where $C^*(t) = c^* e^{\gamma t}$, $A^*(t) = a^* e^{\gamma t}$, $K^*(t) = k^* e^{\gamma t}$, $H^*(t) = h^* = c^*$, such that the endogenous growth rate $\gamma = \gamma_C =$

$\gamma_K = \gamma_A$, c^*, a^* and N^* are jointly determined from the steady state solution of the problem for the centralized economy.

Given the curvature properties of the utility function and of the equation for the accumulation of the natural resource, the first order conditions are both necessary and sufficient.

The current-value Hamiltonian is

$$H(...) = U(C, H, N) + \lambda_1 [F(, N, K) - C - A] + +\lambda_2 [\theta N - p(K, A)] + \lambda_3 [\rho (C - H)] \tag{21}$$

From the maximum principle of Pontryiagin, the optimal levels for consumption and abatement will verify the following first order conditions for an optimum,

$$\begin{aligned} U_C(.) &= \lambda_1 - \rho\lambda_3 && (22) \\ \lambda_1 &= -p_A\lambda_2 && (23) \\ \dot{\lambda}_1 &= (\delta - F_K(.))\,\lambda_1 + p_K\lambda_2 && (24) \\ \dot{\lambda}_2 &= (\delta - n_N)\,\lambda_2 - F_N\lambda_1 - u_N && (25) \\ \dot{\lambda}_3 &= (\rho + \delta)\,\lambda_3 - u_H && (26) \end{aligned}$$

plus the three equations of motion for the three stocks of the economy (14), (19) and (4), and where λ_1, λ_2 and λ_3 are the co-state variables associated to these three stocks. The following transversality conditions

$$\begin{aligned} \lim_{t\to\infty} e^{-\delta t}\lambda_1(t)K(t) &= 0 \\ \lim_{t\to\infty} e^{-\delta t}\lambda_2(t)N(t) &= 0 \\ \lim_{t\to\infty} e^{-\delta t}\lambda_3(t)H(t) &= 0 \end{aligned}$$

hold as we assume that $\delta > 0$.

If all the assumptions hold, then the long-run endogenous growth rate for this economy is

$$\gamma = \frac{1}{\sigma}\left[B\,(N^*)^{\varepsilon} - \delta - a^*\right] \tag{27}$$

and the steady state values for the variables verify

$$N^* = \frac{1}{a^*} \tag{28}$$

and the following system

$$a_{\dot{N}=0} = \frac{1}{\theta N \left[1 - \frac{N}{N_{CC}}\right]} > 0 \tag{29}$$

$$a_{\dot{a}=0} = \sqrt{\frac{\varepsilon B N^{\varepsilon-1} + \frac{\beta}{\alpha\left(1+\frac{\rho}{\rho+\delta}\varphi\frac{c}{h}\right)}\frac{c}{N}}{c - \theta\left(1 - \frac{1}{2}\frac{N}{N_{CC}}\right)}} > 0 \quad \text{for } n_N < c \tag{30}$$

Due to the properties assumed for the equations, the system may generate zero, one or two solutions for the equilibrium level of natural capital, N^* and of the share of abatement activities in capital stock, $\overline{a}$. This economy will have only one solution iff

$$\frac{\partial a_{\dot{a}=0}}{\partial N} < \frac{\partial a_{\dot{N}=0}}{\partial N} \tag{31}$$

and (eventually) two solutions when

$$\frac{\partial a_{\dot{a}=0}}{\partial N} > \frac{\partial a_{\dot{N}=0}}{\partial N} \tag{32}$$

Figure 1 summarizes the general framework of the solution in the (a, N) phase-plane.

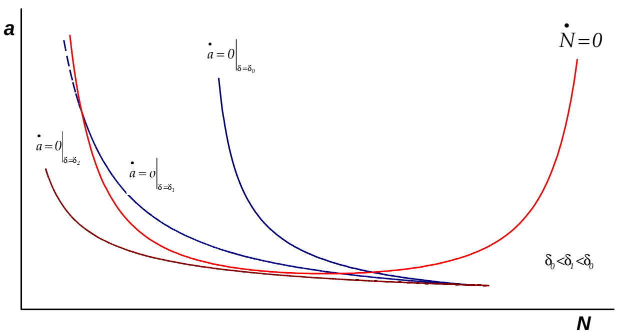

Figure 1: The existence of a zero, one or two solutions depends on the value of the rate of discount.

The existence of a zero, one or two steady-states depends on the value of the rate of discount. Independently of the number of solutions, the balanced growth path is characterized by a constant level of natural capital stock and of the net flow of pollutant emissions. However, the economic variables grow at the same and constant rate but this rate is different for each equilibrium level of environmental quality. The poorer the environmental quality is, the lower will be the long term growth.

On the other hand, the steady state equilibrium values for the economic and natural variables depend on the parameters of the model.

$$N^* = n(\delta, \alpha, \varphi, \rho, \beta, B, \theta) \tag{33}$$
$$a^* = a(\delta, \alpha, \varphi, \rho, \beta, B, \theta) \tag{34}$$
$$\gamma = g(\delta, \alpha, \varphi, \rho, \beta, B, \theta) \tag{35}$$

From the implicit function theorem, it is easy to see that the impacts of the preferences, technological and environmental parameters over the equilibrium values for natural capital, abatement activities (or on the net flow of pollutant emissions, p) and on the long-run growth rate are summarized in the following tables

Parameters		**Impact on N***			
		One SS		Two SS	
		$N^* < N_M$	$N^* > N_M$	$N^* < N_M$	$N^* > N_M$
Preferences	δ	-	-	+	-
	α	-	-	+	-
	φ	-	-	+	-
	ρ	-	-	+	-
	β	+	+	-	+
Technological	B	+	+	-	+
Natural	θ	+	+	-	+

a)

Parameters		**Impact on a***			
		One SS		Two SS	
		$N^* < N_M$	$N^* > N_M$	$N^* < N_M$	$N^* > N_M$
Preferences	δ	+	-	-	-
	α	+	-	-	-
	φ	+	-	-	-
	ρ	+	-	-	-
	β	-	+	+	+
Technological	B	-	+	+	+
Natural	θ	-	+	+	+

b)

Parameters		**Impact on γ**			
		One SS		Two SS	
		$N^* < N_M$	$N^* > N_M$	$N^* < N_M$	$N^* > N_M$
Preferences	δ	-	-	+	-
	α	-	-	+	-
	φ	-	-	+	-
	ρ	-	-	+	-
	β	+	+	-	+
Technological	B	+	+	-	+
Natural	θ	+	+	-	+

Table 1: Impact over the equilibrium level of; **a)** the natural capital stock N^* (panel a)), **b)** the share of abatment expenditures on economic capital a^*(panel b)), and, **c)** the value of the long-run growth rate of the economy γ(panel c)) due to a shift of the preferences, technological and natural parameters. The "plus" ("minus") means that changes in the same (oposite) direction as the change in the specific parameter.

In particular, the presence of habits in relation to the consumption of manufactured good (which is measured by a positive value of ρ and/or φ) lowers the long run equilibrium level of natural capital and the growth rate.

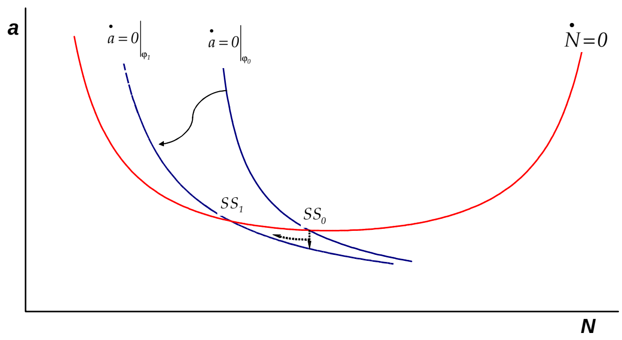

Figure 2: An increase of the strength of habits (ρ) or its relevance for well-being (φ) will reduce the levels of natural capital and pollutant emissions and the long term growth rate associated to a new long term path (SS_1).

The intuition for this effect is straightforward.

First note that an increase of the strength of habits (ρ) or its relevance for well-being (φ) will raise the "overweight" of consumption in utility which, "*coeteris paribus*", will force the decrease of the share of abatement expenditures in economic capital. This can be seen as a shift to the left of the $\dot{a} = 0$ locus on figure 2. As a consequence, the net flow of pollutant emissions will increase and eventually exceed the natural regenerative and assimilative capacity, thus forcing the reduction of the stock of natural capital.

However, in this region, the natural regenerative capacity is increasing with the stock of natural assets which increases the need of additional abatement activities in the economy to assure the fulfilment of the sustainability condition $\dot{N} = 0$. The new steady-state (SS_1) is then characterized by lower levels of natural capital and pollutant emissions. On the other hand, the economy is now in a new long term path that has associated a new trend growth rate that is less then the one related to the initial SS_0.

An increase of the strength of habits (ρ) or its relevance for well-being (φ) will reduce the levels of natural capital and pollutant emissions and the long-run growth rate associated to a new long term path (SS_1).

When the solution is located at the ascending arm of the $\dot{N} = 0$, the new steady-state is characterized by a lower level of natural capital but conversely to the previous case, by a higher flow of net pollutant emissions. This is due to the fact that in this region of high level of natural capital, most of it is used in the regenerative processes and only a small amount of energy is available for pollution absorption (see Smulders (1995)). In other words, for high levels of environmental quality, natural regenerative and assimilative capacity is decreasing with the stock of natural capital which means that any reduction of this stock will increase the assimilative capacity. Therefore, the reduction of environmental quality will be followed by an increase in the natural regenerative and assimilative activity which reduces the need for additional abatement activities.

On the other hand, the model allows for "win-win" solutions. Whenever only one steady-state exists and independently of its location, any change of environmental preferences towards a more environmental concern (which is modelled as an increase of β) will result in an increase of environmental quality and of the long term growth rate. However, the impact over the net flow of pollutant emissions depends on the level of the natural capital associated with the initial SS. For initially poor environmental quality, a permanent increase of the amenity value of the environment will cause an increase in the emissions associated with the new SS because the resulting improvement of the stock of natural capital is followed by an increase of the natural regenerative and assimilative capacity. The opposite occurs for relatively high levels of natural capital.

Finally, the model also shows that the presence of habits influences the outcome of changes in the amenity value of the environment over the existing levels of natural capital, pollutant emissions and on the long term growth rate. In particular, the stronger the habits are over well-being the lesser will be the equilibrium levels of natural capital and the share of income devoted to pollution-abatement. On the other hand, the increase of consumption inertia will also reduce the level of the long term growth rate associated with the new time path for the economy.

5 Conclusions

The paper presents a model of endogenous growth models with habit formation in relation to manufactured consumption goods. We show that there may be multiple equilibria. If it exists, the "*optimal sustainable balanced growth*" means a path in which the "economic variables" (output, consumption, man-made capital) grow at a positive rate, while the "environmental variables" (flow of pollution, the stock of environmental capital or the environmental quality) remain constant over time. So, if the economy grows along this balance growth path in the long run, the economy is also characterized by sustainable development as defined by the most cited definition of sustainability. However, the long term growth rate is different for each equilibrium level of environmental quality. The poorer the environmental quality is, the lower will be the long term growth rate.

The steady state equilibrium values for the economic and natural variables depend on the parameters of the model. We present a complete characterization of the effects of the parameters over the steady-state values for natural capital stock, for the share of abatement expenditures in economic capital and for the endogenous growth rate. In particular, and in contrast with the few approaches that deal with habit-formation, sustainability and growth, we show that the presence of habits in relation to the consumption manufactured goods reduces the optimal level of natural capital and the growth rate associated with the long term time path of the economy.

Moreover, we also show that presence of habits influences the outcome of changes in the amenity value of the environment over the existing levels of natural capital, pollutant emissions and on the long term growth rate. The stronger the inertia effect caused by habits, the lesser will be the equilibrium levels of natural capital and the higher will be the net flow of pollutant emissions. At the same time, the economy will grow at a lower rate.

Furthermore, the paper also show that "win-win" solutions are possible, that is, an increase in environmental quality and an increase in the long term growth rate as a result a change in the amenity value of natural capital.

Finally we would like to highlight that this work is a first and preliminary output of research devoted to the effect that the process of consumption inertia might have over the economic-environmental relations and specifically to sustainable endogenous growth. Taking this in mind, several extensions are possible and already identified.

One extension is the full characterization of the stability conditions and the possible transitional dynamics associated to each steady-state. We suspect that cyclical behavior, stable and/or unstable, might appear due to the presence of habit- persistence like most research predicts.

The second extension is the consideration of habits in relation to natural capital goods instead of manufactured consumption goods. Given the present civilizational and cultural contour of preferences, consumers have to endure a learning-by-consuming (or a habit-formation) process to full enjoy (and use) them and to undertake a "green-economic behavior". This “learning-by consuming” process is equivalent to “adjacent complementarity”. Naturally, the stock of environmental quality is also a “beneficial addiction” as it has a positive value for the consumer. What the consequences are for the sustainability and growth concerns of the presence of this “learning-by-consuming” process, will be the extension of the present research.

6 Appendix A

Consider the Hamiltonian system of the optimum control problem

$$\begin{aligned}
\dot{K} &= F(K,N) - C - A && \text{(A.1)}\\
\dot{N} &= n(N) - \frac{1}{a} && \text{(A.2)}\\
\dot{H} &= \rho\,[C - H] && \text{(A.3)}\\
\dot{\lambda}_1 &= (\delta - F_K(.))\,\lambda_1 + p_K\lambda_2 && \text{(A.4)}\\
\dot{\lambda}_2 &= (\delta - n_N)\,\lambda_2 - F_N\lambda_1 - u_N && \text{(A.5)}\\
\dot{\lambda}_3 &= (\rho + \delta)\,\lambda_3 - u_H && \text{(A.6)}
\end{aligned}$$

Equating (36) to zero, using equation (36), the two first order conditions (22) and (23) and considering that $\frac{\dot{a}}{a} = \frac{\dot{A}}{A} - \frac{\dot{K}}{K}$, we get the following expression for the ratio $a = \frac{A}{K}$;

$$\dot{a} = \frac{a}{2}\left\{(c - n_N) - \left(F_N + \frac{u_N}{u_C + \frac{\rho}{\rho+\delta}u_H}\right)\frac{a^{-2}}{K}\right\} \tag{36}$$

Equating this expression to zero we get (30).

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